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On perfectly $\alpha\delta$-continuous functions in topological spaces

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Abstract

The concept of $\alpha\delta$-closed sets was introduced in the research paper "On Strongly-αδ-Super-Irresolute Functions In Topological Spaces. The aim of this paper is to consider and characterize $\alpha\delta$-irresolute and $\alpha\delta$-continuous functions via the concept of $\alpha\delta$-closed sets and to relate these concepts to the classes of $\alpha\delta O$-compact spaces and $\alpha\delta$-connected spaces.

Keywords: αδ-irresolute functions, αδ-continuous functions, αδO-compact spaces, αδ-connected spaces.

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1.INTRODUCTION

Closedness is a basic concept for the study and investigation of topological spaces. This concept has been generalized and studied by many authors from different points of views. In particular,Introduced semi-open sets, pre-open sets, α-open sets, δ-closed sets and δ-semi-closed sets, gpr-closed sets respectively.[1-8]Introduced α-generalized closed (briefly αg-closed) sets.[9],Introduced and studied the concept of δg-closed sets, δgs-closed sets and δsg-closed sets.[5,10-11] More recently,Introduced and studied the notion of $\alpha\delta$-closed sets which is implied by that of δ-closed sets and implies that of $g\delta s$-closed sets.[12]

The notions of $\alpha\delta$-open sets, $\widetilde{T_{\alpha\delta}}$ -spaces, $\alpha\delta$-continuity and $\alpha\delta$-irresoluteness are also available. In this paper, we will continue the study of $\alpha\delta$-closed sets and associated function with introducing and characterizing $\alpha\delta$- continuous and $\alpha\delta$-irresolute functions. Further, we introduce the concepts of strong - αδ- continuity, perfect - αδ- continuity, $\alpha\delta O$-compactness and $\alpha\delta$-connectedness, and study their behaviour under $\alpha\delta$-continuous functions.

Throughout this paper, spaces X and Y always mean topological spaces. Let X be a topological space and A bea subset of X. The closure of A and the interior of A are denoted by $cl(A)$ and $int(A)$, respectively. A subset $A$is said to be regular open (resp. regular closed) if $A = int(cl(A))$, (resp. $A = cl(int(A))$, The δ-interior [4] of a subset $A$of $X$is the union of all regular open sets of Xcontained in $A$and is denoted by $Int_\delta(A)$. The subset A is called δ-open [4] if $A = Int_\delta(A)$, *i.e.,* a set is δ-open if it is the union of regular open sets. The complement of a δ-open set is called δ-closed.

Alternatively, a set $A \subset (X, \tau)$ is called δ-closed [4] if $A = cl_\delta(A$,where $cl_\delta(A) = \{x \mid x \in U \in \tau \Rightarrow int(cl(U)) \cap A \neq \emptyset\}$. The family of all δ-open (resp. δ-closed) sets in $X$is denoted by $\delta O(X)$ (resp. $\delta C(X)$). A subset A of X is called semi-open[1] (resp. α-open [8],δ-semi-open [5] if $A \subset cl(int(A))$ (resp.$A \subset int(cl(int(A)))$, $A \subset cl(Int_\delta(A))$ and the complement of a semi-open(resp. α-open, δ-semi-open) is called semi-closed(resp. α-closed, δ-semi-closed). The intersection of all semi-closed (resp. α-closed, δ-semi-closed) sets containing A is called the semi-closure(resp. α-closure, δ-semi-closure) of A and is denoted by $scl(A)$(resp.$\alpha cl(A)$, δ-$scl(A)$). Dually, semi-interior (resp. α-interior, δ-semi-interior) of A is defined to be the union of all semi-open (resp. α-open,δ-semi-open) sets contained in A and is denoted by $sint(A)$(resp.$\alpha int\ (A)$, δ-$sint(A)$). Note that δ-$scl(A) = A \cup int(cl_\delta(A))$ and δ-$sint(A)$)$= A \cup cl(Int_\delta(A))$. A is $en(= closed\ and\ open)$.

We recall the following definition used in the sequel.

Definition 1.1. A subset A of a space X is said to be

(a) An α-generalized closed [9] (αg-closed) set if $\alpha cl(A) \subseteq U$whenever $A \subseteq U$ and U is α-open in (X, τ).
(b) An $\alpha\delta$-closed [12] set if $cl_\delta(A) \subseteq U$whenever $A \subseteq U$ and U is αg-open in (X, τ).
(c) δ-semi-generalized closed [13] (δsg-closed) if δ-$scl(A) \subseteq U$whenever $A \subseteq U$ and U is open in (X, τ).
(d) A generalized δ-semi-closed [13] ($g\delta s$-closed) set if δ-$scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in(X, τ).
(e) A δ-generalized closed [10] (δg-closed) set if $cl_\delta(A) \subseteq U$whenever $A \subseteq U$ and U is open in (X, τ).

Remark 1.2.[13] The following diagram holds for a subset A in a topological space (X, τ):

δg-closed$\longrightarrow$ δsg-closed
$\uparrow$ $\uparrow$
δ-closed$\longrightarrow$ δ-semiclosed

Definition 1.3. *Cantor-Bendixson derivative* **[14]**

Let A be a subset of a topological space X. Its Cantor-Bendixson derivative A' is defined as the set of accumulation points of A. In other words $A' = \{x \in X | x \in \overline{A\backslash\{x\}}\}$. Through transfinite induction, the Cantor-Bendixson derivative can be defined to any order α, where α is an arbitrary ordinal. Let $A^{(0)} = A$. If $\alpha$is a successor ordinal, then $A^{(\alpha)} = (A^{(\alpha-1)})'$**.** If λ is a limit ordinal, then $A^{(\lambda)} = \bigcap_{\alpha<\lambda} A^{(\alpha)}$. The Cantor-Bendixson rank of the set A is the least ordinal α such that $A^{(\alpha)} = A^{(\alpha+1)}$. Note that $A' = A$implies that A is a perfect set.

2.$\alpha\delta$-Interior and $\alpha\delta$-Closure

Definition 2.1. Let X be a topological space and let $x \in X$. A subset N of X is said to be an$\alpha\delta$-nbhd of x if there exists an$\alpha\delta$-open set G such that $x \in G \subset N$.

Definition 2.2. Let A be a subset of X. A point $x \in A$ is said to be $\alpha\delta$-interior point of A if A is an$\alpha\delta$-nbhd of x. The set of all $\alpha\delta$-interior point of A is called the $\alpha\delta$-interior of A and is denoted by $\alpha\delta_{Int}(A)$.

Definition 2.3. For a subset A of (X,τ), we define the $\alpha\delta$-closure of A as follows:

$$\alpha\delta_{Cl}(A) = \bigcap\{F: F \text{ } is \text{ } \alpha\delta - closed \text{ } in \text{ } X, \text{ } A \subset F\}.$$

Lemma 2.4. Let A be a subset of $(X,\tau)$and $x \in X$. Then $x \in \alpha\delta_{Cl}(A)$ if and only if $V \cap A \neq \emptyset$ for every $\alpha\delta$-open set Vcontaining x.

Proof: Suppose that there exists a $\alpha\delta$-open set V containing x such that $V \cap A = \emptyset$. Since $A \subset X\backslash V$, $\alpha\delta_{Cl}(A) \subset X\backslash V$ and then $x \notin \alpha\delta_{Cl}(A)$, a contradiction. Conversely, suppose that $x \notin \alpha\delta_{Cl}(A)$. Then there exist an$\alpha\delta$-closed set F containing A such that $x \notin F$. Since $x \in X\backslash F$ and $X\backslash F$ is $\alpha\delta$-open,$(X\backslash F) \cap A = \emptyset$, a contradiction.

Lemma 2.5. Let A and B be subsets of (X,τ). Then we have

(a) $\alpha\delta_{Cl}(\emptyset) = \emptyset$and $\alpha\delta_{Cl}(X) = X$.
(b) If A is $\alpha\delta$-closed, then $\alpha\delta_{Cl}(A) = A$.
(c) If $A \subset B$, then $\alpha\delta_{Cl}(A) \subset \alpha\delta_{Cl}(B)$.
(d) $\alpha\delta_{Cl}(A) = \alpha\delta_{Cl}(\alpha\delta_{Cl}(A))$.

Proof: Straightforward.

Lemma 2.6. Let (X,τ) be a topological space and $A \subset X$, $B \subset X$. The following properties hold:

(a) $\alpha\delta_{Int}(\emptyset) = \emptyset$ and $\alpha\delta_{Int}(X) = X$.
(b) $\alpha\delta_{Int}(A) \subset A$.
(c) $\alpha\delta_{Int}(A) = \alpha\delta_{Int}(\alpha\delta_{Int}(A))$.
(d) If $A \subset B$, then $\alpha\delta_{Int}(A) \subset \alpha\delta_{Int}(B)$.
(e) If B is any $\alpha\delta$-open set contained in A, then $B \subset \alpha\delta_{Int}(A)$.

Proof: Straightforward.

Remark 2.7. (a) If A is $\alpha\delta$-closed in (X,τ), then $\alpha\delta_{Cl}(A) = A$. But the converse is not true which shows from the following example: Let $X = \{a,b,c,d\}$ and $\tau = \{X, \emptyset, \{a\}, \{b\}, \{d\}, \{a,b\}, \{a,d\}, \{b,d\}, \{a,b,d\}\}$.

Let $A = \{b,d\}$.Then $\alpha\delta_{Cl}(A) = A$but A is not $\alpha\delta$-closed.

(b) In general, $\alpha\delta_{Cl}(A \cap B) \not\supset \alpha\delta_{Cl}(A) \cap \alpha\delta_{Cl}(B)$. For example, let $X = [-1,1]$ and τ be the uncountable fort topology on X, *i.e.*,$\tau = \{U \subset [-1,1]: 0 \notin U \text{ } or \text{ } X\backslash U \text{ } is \text{ } finite\}$. Let $A = (-1,0)$and $B = (0,1)$. Since both A and B are not $\alpha\delta$-closed, $\alpha\delta_{Cl}(A) = (-1,0]$ and $\alpha\delta_{Cl}(B) = [0,1)$ and so $\alpha\delta_{Cl}(A) \cap \alpha\delta_{Cl}(B) = \{0\}$ but $\alpha\delta_{Cl}(A \cap B) = \emptyset$.

Theorem 2.8. Let A be any subset of X. Then

(a) $(\alpha\delta_{Int}(A))^c = \alpha\delta_{Cl}(A^c)$
(b) $\alpha\delta_{Int}(A) = (\alpha\delta_{Cl}(A^c))^c$
(c) $\alpha\delta_{Cl}(A) = (\alpha\delta_{Int}(A^c))^c$.

Proof: (a) Let $x \in (\alpha\delta_{Int}(A))^c$. Then $x \notin \alpha\delta_{Int}(A)$, i.e., every $\alpha\delta$-open set U containing x is such that $U \not\subset A$. Then every $\alpha\delta$-open set U containing x is such that $U \cap A^c \neq \emptyset$. By Lemma 2.4, $x \in \alpha\delta_{Cl}(A^c)$ and therefore $(\alpha\delta_{Int}(A))^c \subset \alpha\delta_{Cl}(A^c)$. Conversely, let $x \in \alpha\delta_{Cl}(A^c)$. Then by Lemma 2.4, every $\alpha\delta$-open set U containing x is such that $U \cap A^c \neq \emptyset$.

That is every $\alpha\delta$-open set U containing x is such that $U \not\subset A$. This implies by definition of $\alpha\delta$-interior of A, $x \notin \alpha\delta_{Int}(A)$. That is $x \in (\alpha\delta_{Int}(A))^c$and $\alpha\delta_{Cl}(A^c) \subset (\alpha\delta_{Int}(A))^c$. Thus $(\alpha\delta_{Int}(A))^c = \alpha\delta_{Cl}(A^c)$.

(b) Follows by taking complements in (a).

(c) Follows by replacing A by A^c in (a).

Definition 2.9. For a subset A of (X,τ)

(a) $Int_\delta(A) = \bigcup\{F: F is \delta - open in X, F \subset A\}$[4].
(b) δgs-$int(A) = \bigcup\{F: F is \delta gs - open in X, F \subset A\}$[5].
(c) δsg-$int(A) = \bigcup\{F: F is \delta sg - open in X, F \subset A\}$[12].

Theorem 2.10. If A is a subset of X, Then

$\alpha\delta_{Int}(A) = \bigcup\{F: F \text{ } is \text{ } \alpha\delta - open \text{ } in \text{ } X, F \subset A\}$.

Proof: IfA be a subset of X, Then

$x \in \alpha\delta_{Int}(A) \Leftrightarrow x$ is a $\alpha\delta$-interior point of A.
$\Leftrightarrow A$ is a $\alpha\delta$-nbhd of point x.
$\Leftrightarrow$ there exists $\alpha\delta$-open set F such that $x \in F \subset A$.
$\Leftrightarrow x \in \bigcup\{F: F \text{ } is \text{ } \alpha\delta - open \text{ } in \text{ } X, F \subset A\}$.
$\alpha\delta_{Int}(A) = \bigcup\{F: F \text{ } is \text{ } \alpha\delta - open \text{ } in \text{ } X, F \subset A\}$.

Theorem 2.11. If A is a subset of X, Then

(a) $Int_\delta(A) \subset \alpha\delta_{Int}(A)$.
(b) $\alpha\delta_{Int}(A) \subset \delta gs$-$Int(A)$.
(c) $\alpha\delta_{Int}(A) \subset g\delta s$-$Int(A)$.

Proof: (a) Let A be a subset of X, Then

$x \in Int_\delta(A) \Rightarrow x \in \bigcup\{F \subset X : F \text{ } is \text{ } \delta - open, F \subset A\}$.
$\Rightarrow$ There exists δ-open set F such that $x \in F \subset A$.
$\Rightarrow$There exists $\alpha\delta$-open set F such that $x \in F \subset A$, as every δ-open set is a $\alpha\delta$-open set in X.
$\Rightarrow x \in \bigcup\{F \subset X : F \text{ } is \text{ } \alpha\delta - open, F \subset A\}$.
$\Rightarrow x \in \alpha\delta_{Int}(A)$.
Thus $x \in Int_\delta(A) \Rightarrow x \in \alpha\delta_{Int}(A)$.
Hence $Int_\delta(A) \subset \alpha\delta_{Int}(A)$.

The proof of (b) and (c) follows from (a).Since we know that every $\alpha\delta$-open set is a δgs-open set and every $\alpha\delta$-open set is a $g\delta s$-open set. Containment relation in the above may be proper as seen from the following example.

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Example 2.12.

(a) Let $X = \{a, b, c\}$ and $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$, $\alpha\delta_{Int}(\{a\}) = \{a\}$ and $Int_\delta(\{a\}) = \emptyset$. It follows that $Int_\delta(\{a\}) \subset \alpha\delta_{Int}(\{a\})$ and $Int_\delta(\{a\}) \neq \alpha\delta_{Int}(\{a\})$.

(b) Let $X = \{a, b, c, d\}$ and $\tau = \{X, \emptyset, \{a\}, \{c\}, \{a, c\}, \{a, c, d\}\}$, $\alpha\delta_{Int}(\{b\}) = \emptyset$ and $\delta gs\text{-}Int(\{b\}) = \{b\}$. It follows that $\alpha\delta_{Int}(\{b\}) \subset \delta gs\text{-}Int(\{b\})$ and $\alpha\delta_{Int}(\{b\}) \neq \delta gs\text{-}Int(\{b\})$.

(c) Let $X = \{a, b, c, d\}$ and $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$, $\alpha\delta_{Int}(\{d\}) = \emptyset$ and $\delta sg\text{-}Int(\{d\}) = \{d\}$. It follows that $\alpha\delta_{Int}(\{d\}) \subset \delta sg\text{-}Int(\{d\})$ and $\alpha\delta_{Int}(\{d\}) \neq \delta sg\text{-}Int(\{d\})$.

Definition 2.13. The union of all πg-open sets, [15] each contained in a set S in a topological space X is called the πg-interior of S and it is denoted by $\pi g\text{-}int(S)$.

Remark 2.14. For a subset A of a space , $\alpha\delta_{Int}(A) \subset \pi g\text{-}int(A)$.

Definition 2.15. Let (X, τ) be a topological space[12] and $S \subset X$. The set $\cap\{A: S \subset A \text{ and } A \in \delta sg - closed\}$ is called δ-semi-generalized closure of S and is denoted by $\delta sg\text{-}cl(S)$.

Definition 2.16. For a topological space (X, τ),

(a) $\tau^*_{\alpha\delta} = \{U \subset X \mid \alpha\delta_{Cl}(X \backslash U) = X \backslash U\}$.

(b) $\tau^*_\delta = \{U \subset X \mid cl_\delta(X \backslash U) = X \backslash U\}$.

(c) $\tau^\#_{\delta sg} = \{U \subset X \mid \delta sg\mathrm{Cl}(X \backslash U) = X \backslash U\}$.

Definition 2.17. A space (X, τ) is called

(a) T^δ-space if every δ-closed set in it is closed.

(b) Submaximal [11] if every dense subset is open.

(c) $\widetilde{T_{\alpha\delta}}$ –space [11] if every $\alpha\delta$-closed set in it is δ-closed.

Proposition 2.18. If $\delta O(X, \tau)$ is closed under finite intersections, then $\tau^*_{\alpha\delta}$ is a topology for X.

Proof: Clearly, $\emptyset, X \in \tau^*_{\alpha\delta}$. Let $\alpha \in I$ and $A_\alpha \in \tau^*_{\alpha\delta}$. Then $\alpha\delta_{Cl}(X \backslash (\cup_\alpha A_\alpha)) = \alpha\delta_{Cl}(\cap_\alpha (X \backslash A_\alpha)) \subset \cap_\alpha \alpha\delta_{Cl}(X \backslash A_\alpha) = \cap_\alpha (X \backslash A_\alpha) = X \backslash \cup_\alpha A_\alpha$ and hence $\cup_\alpha A_\alpha \in \tau^*_{\alpha\delta}$. Let $A, B \in \tau^*_{\alpha\delta}$. Then $\alpha\delta_{Cl}(X \backslash (A \cap B)) = \alpha\delta_{Cl}(X \backslash A) \cup \alpha\delta_{Cl}(X \backslash B) = X \backslash (A \cap B)$ and hence $A \cap B \in \tau^*_{\alpha\delta}$. Therefore $\tau^*_{\alpha\delta}$ is a topology.

Proposition 2.19. For a subset A of (X, τ), the following statements hold:

(a) $A \subset cl_\delta(A) \subset \alpha\delta_{Cl}(A) \subset \delta sg\text{-}cl(A)$.

(b) $\tau^*_\delta \subset \tau^*_{\alpha\delta} \subset \tau^\#_{\delta sg}$.

Proof: The proof follows from the definitions.

Definition 2.20. A subset A of a space X is called generalized δp-regular closed (briefly, $g\delta pr$-closed) [11] if $\delta\text{-}pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in (X, τ).

Definition 2.21. For a subset A of a topological space[11] (X, τ), $g\delta pr\text{-}cl(A) = \cap\{F: A \subset F, F \text{ is } g\delta pr - closed \text{ in } X\}$.

Definition 2.22. The intersection of all πg-closed sets,[16] each containing a set S in a topological space X is called the πg-closure of S and it is denoted by $\pi g\text{-}cl(S)$.

Remark 2.23. For a subset A of a space, $g\delta pr\text{-}cl(A) \subset \alpha\delta_{Cl}(A)$ and $\pi g\text{-}cl(A) \subset \alpha\delta_{Cl}(A)$.

Theorem 2.24. Let (X, τ) be a space. Then

(a) Every $\alpha\delta$-closed set is δ-closed (*i.e.*,(X, τ) is $\widetilde{T_{\alpha\delta}}$ -space) if and only if $\tau^*_{\alpha\delta} = \delta O(X, \tau)$.

(b) Every $\alpha\delta$-closed set is closed if and only if $\tau^*_{\alpha\delta} = \tau$.

(c) If every δsg-closed set is $\alpha\delta$-closed (*i.e.*, $G\delta sC(X, \tau) = \alpha\delta C(X, \tau)$), then $\tau^*_{\alpha\delta} = \tau^\#_{\delta sg}$.

Proof. (a) Let $A \in \tau^*_{\alpha\delta}$. Then $\alpha\delta_{Cl}(X \backslash A) = X \backslash A$. By hypothesis, $cl_\delta(X \backslash A) = \alpha\delta_{Cl}(X \backslash A) = X \backslash A$ and hence $A \in \delta O(X, \tau)$. Conversely, let A be a $\alpha\delta$-closed set. Then $\alpha\delta_{Cl}(A) = A$ and hence $X \backslash A \in \tau^*_{\alpha\delta} = \delta O(X, \tau)$, *i.e.*, A is δ-closed.

(b) Similar to (a).

(c) Let $A \in \tau^\#_{\delta sg}$. Then δsg-cl$(X \backslash A) = X \backslash A$ and by hypothesis, $\alpha\delta_{Cl}(X \backslash A) = \delta sg$-cl$(X \backslash A) = X \backslash A$.

Hence $\in \tau^*_{\alpha\delta}$, *i.e.*,$\tau^\#_{\delta sg} \subset \tau^*_{\alpha\delta}$. By Proposition 2.19, $\tau^*_{\alpha\delta} = \tau^\#_{\delta sg}$.

Theorem 2.25. Given a space X, it follows that

(a) If $\tau^*_{\alpha\delta} = \delta O(X)$ then X is submaximal and $\widetilde{T_{\alpha\delta}}$-space.

(b) If X is submaximal and $\widetilde{T_{\alpha\delta}}$-space then X is T^δ-space and $\widetilde{T_{\alpha\delta}}$-space.

Proof. (a) Let A be a δ-open subset of X. By Proposition 2.19, we have $A \in \tau^*_{\alpha\delta}$. By $\tau^*_{\alpha\delta} = \delta O(X)$, A is open. Hence X is submaximal. Let B be a $\alpha\delta$-closed subset of X. By Theorem 2.24(a), B is δ-closed and so $\alpha\delta$-closed. Hence X is $\widetilde{T_{\alpha\delta}}$-space.

(b) It is proved in. [8]

3. Strongly αδ-continuous and Perfectly αδ-continuous functions.

Definition 3.1. A function[11] $f: (X, \tau) \to (Y, \sigma)$ is called

(a) αδ-continuous if $f^{-1}(V)$ is an αδ-closed set in (X, τ) for every closed set V of (Y, σ).

(b) αδ-irresolute if $f^{-1}(V)$ is an αδ-closed set in (X, τ) for every αδ-closed set V of (Y, σ).

(c) Strongly-$\alpha\delta$-super-Irresolute (briefly, $S\alpha\delta$-super-Irr) if $f^{-1}(V)$ is aδ-closed set in (X, τ) for every αδ-closed set V of (Y, σ).

(d) Super-αδ-continuous if $f^{-1}(V)$ is αδ-closed set in (X, τ) for every δ-closed set V of (Y, σ).

(e) Super-continuous [Noiri T, (1979/80)] if $f^{-1}(V)$ is δ-closed set in (X, τ) for every closed set V of (Y, σ).

Remark 3.2.

(a) If $\tau^*_{\alpha\delta} = \delta O(X, \tau)$ in X, then Super-continuity and $\alpha\delta$-continuity coincide.

(b) Every $\alpha\delta$-continuous function defined on a $\widetilde{T_{\alpha\delta}}$-space is Super-continuous.

Definition 3.3. A subset A[11] of a space (X, τ) is said to be $g\delta p$-closed) if $\delta\text{-}pcl(A) \subseteq U$ whenever $A \subseteq U$ and $U \in \tau$. The compliment of $g\delta p$-closed set is said to be $g\delta p$-open.

Definition 3.4. A subset A [5] of a space (X, τ) is said to be πgp-closed) if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is π-open in X.

Definition 3.5. A function[11] $f: (X, \tau) \to (Y, \sigma)$ is called $g\delta p$-continuous (resp. $g\delta pr$-continuous) if $f^{-1}(F)$ is $g\delta p$-closed (resp. $g\delta pr$-closed) in X for every closed set F of Y.

Definition 3.6. A function[15] $f: (X, \tau) \to (Y, \sigma)$ is said to be almost πgp-continuous if $f^{-1}\left(int\left(cl(V)\right)\right)$ is πgp-open for every $V \in \sigma$.

Remark 3.7. Every $\alpha\delta$-continuous function is $g\delta p$-continuous and so $g\delta pr$-continuous and almost πgp-continuous.

Theorem 3.8. Let $f: (X, \tau) \to (Y, \sigma)$ be a function.

(a) The following statements are equivalent:

(i) $f$is $\alpha\delta$-continuous.

(ii) The inverse image of every open set in Y is $\alpha\delta$-open in$X$.

(b) If $f:(X,\tau)\to(Y,\sigma)$ is $\alpha\delta$-continuous, then $f(\alpha\delta_{Cl}(A))\subset cl(f(A))$ for every subset A of X.

(c) The following statements are equivalent:

(i) For each $x\in X$ and each open set Vcontaining $f(x)$, there exists a $\alpha\delta$-open set$U$ containing $x$such that $f(U)\subset V$.

(ii) For every subset A of X,$f(\alpha\delta_{Cl}(A))\subset cl(f(A))$.

(iii)Suppose $\tau^*_{\alpha\delta}$ is a topology. The function $f:(X,\ \tau^*_{\alpha\delta})\to(Y,\sigma)$ is continuous.

Proof:

(a) Straightforward.

(b) Let $A\subset X$. Since $f$is $\alpha\delta$-continuous and $A\subset f^{-1}(cl(f(A)))$, we obtain $\alpha\delta_{Cl}(A)\subset f^{-1}(cl(f(A)))$ and then $f(\alpha\delta_{Cl}(A))\subset cl(f(A))$.

(c) $(i)\Rightarrow(ii)$ Let $y\in f(\alpha\delta_{Cl}(A))$ and let V be any open neighborhood of y. Then there exists an$x\in X$ and a $\alpha\delta$-open set U such that $f(x)=y, x\in U, x\in\alpha\delta_{Cl}(A)$and $f(U)\subset V$. By lemma 2.4.$U\cap A\neq\emptyset$and hence $f(A)\cap V\neq\emptyset$. Hence $y=f(x)\in cl(f(A))$.

$(ii)\Rightarrow(i)$ Let $x\in X$ and V be any open set containing$f(x)$. Let $A=f^{-1}(Y/V)$. Since $f\left(\alpha\delta_{Cl}(A)\right)\subset cl(f(A))\subset Y/V$, then

$\alpha\delta_{Cl}(A)=A$. Since $x\notin\alpha\delta_{Cl}(A)$, there exists a $\alpha\delta$-open set U containing x such that $U\cap A=\emptyset$ and hence $f(U)\subset\ f(X/A)\subset V$.

$(ii)\Rightarrow(iii)$ Let B be closed in(Y,σ). By hypothesis, $f(\alpha\delta_{Cl}(f^{-1}(B)))\subset cl\left(f\left(f^{-1}(B)\right)\right)\subset cl(B)=B$,*i.e.*,$\alpha\delta_{Cl}(f^{-1}(B))\subset B$. This implies that $f^{-1}(B)$ is closed in $(X,\ \tau^*_{\alpha\delta})$ and hence $f:(X,\ \tau^*_{\alpha\delta})\to(Y,\sigma)$ is continuous.

$(iii)\Rightarrow(ii)$ Let A be any subset of X. Since $cl\big(f(A)\big)$ is closed in (Y,σ).Then the function$f:(X,\ \tau^*_{\alpha\delta})f:(X,\ \tau^*_{\alpha\delta})\to(Y,\sigma)$is continuous, $f^{-1}\left(cl\big(f(A)\big)\right)$is closed in $(X,\ \tau^*_{\alpha\delta})$ and hence $\alpha\delta_{Cl}(f^{-1}(cl(f(A))))=f^{-1}\left(cl\big(f(A)\big)\right)$.Also since $A\subset f^{-1}\big(f(A)\big)\subset f^{-1}\left(cl\big(f(A)\big)\right)$,byLemma 2.5, we have $\alpha\delta_{Cl}(A)\subset\alpha\delta_{Cl}\left(f^{-1}\left(cl\big(f(A)\big)\right)\right)=f^{-1}\left(cl\big(f(A)\big)\right)$ and so $f(\alpha\delta_{Cl}(A)\subset cl\big(f(A)\big)$.

R.Devi, V.Kokilavani and P.Basker pointed out that the composition of two $\alpha\delta$-continuous functions need not be $\alpha\delta$-continuous. However, we have the following.

Proposition 3.9. Let (Y,σ) be a space such that $\sigma^*_{\alpha\delta}=\sigma$. If $f:(X,\tau)\to(Y,\sigma)$ and $g:(Y,\sigma)\to(Z,\eta)$ are $\alpha\delta$-continuous functions, then $g\circ f:(X,\tau)\to(Z,\eta)$is also $\alpha\delta$-continuous.

Proof. It follows from Theorem 2.24(b).

W.r.to. the restriction of a $\alpha\delta$-continuous function, we have the following.

Proposition 3.10. Let $f:(X,\tau)\to(Y,\sigma)$ be a $\alpha\delta$-continuous function and H be a αg-open $\alpha\delta$-closed subset of X. Assume that $\alpha\delta C(X,\tau)$ is closed under finite intersections. Then the restriction $f|H:(X,\tau|\mathrm{H})\to(Y,\sigma)$ is $\alpha\delta$-continuous.

Proof. Let F be a closed subset of Y. By the hypothesis, $f^{-1}(F)\cap H=H_1$(say) is $\alpha\delta$-closed in X. Since $(f|H)^{-1}(F)=H_1$, it is sufficient to show that H_1 is $\alpha\delta$-closed in H. Let G_1 be any αg-open set of H such that $H_1\subset G_1$. $G_1=G\cap H$ for some αg-open set of X. Since $H_1\subset G$ and H_1 is $\alpha\delta$-closed in X, then $\delta cl_X(H_1)\subset G$, $\delta cl_H(H_1)=\delta cl_X(H_1)\cap H\subset G\cap H=\ G_1$ and so H_1 is $\alpha\delta$-closed in H. Hence $f|H$ is $\alpha\delta$-continuous.

Theorem 3.11. For a function$f:(X,\tau)\to(Y,\sigma)$, the following implications hold:

$S\alpha\delta$-super-Irr⇒ αδ-irresolute⇒ Super-αδ-continuous.

However, the converse implications in the theorem above are not always true, as can be seen from the following examples.

Example 3.12. (a) Let $X=\{a,b,c\}$ and$\tau=\{X,\varphi,\{a\},\{b\},\{a,b\},\{a,c\}\}$ and$\sigma=\{X,\varphi,\{a\},\{b\},\{a,b\}\}$. Let $f:(X,\tau)\to(X,\sigma)$ be the identity. Then $f^{-1}(\{b,c\})$ is αδ-irresolute but not a $S\alpha\delta$-super-Irr,since $f^{-1}(\{b,c\})$ is not a δ-closed set in(X,τ).

(b) Let $X=\{a,b,c\}$and$\tau=\{X,\varphi,\{b\},\{c\},\{b,c\},\{c,a\}\}$ and$\sigma=\{X,\varphi,\{a\},\{b\},\{a,b\},\{a,c\}\}$. Let $f:(X,\tau)\to(X,\sigma)$ be the identity. Then $f^{-1}(\{c\})$ is Super-$\alpha\delta$-continuous but not an$\alpha\delta$-Irresolute. Since $f^{-1}(\{c\})$ is not an$\alpha\delta$-closed set in (X,τ).

Definition 3.13.A function $f:(X,\tau)\to(Y,\sigma)$ is called Strongly-αδ-continuous (briefly,ST-$\alpha\delta$-Continuous) if $f^{-1}(V)$ is an open set in (X,τ) for every αδ-open set V of(Y,σ).

Definition 3.14. A function $f:(X,\tau)\to(Y,\sigma)$ is called Perfect-αδ-continuous (briefly, PER-$\alpha\delta$-Continuous) if $f^{-1}(V)$ is clopen set in (X,τ) for every αδ-open set$V$ of(Y,σ).

Note that a function $f:(X,\tau)\to(Y,\sigma)$ is Perfect-$\alpha\delta$-continuous if and only if the inverse image of every αδ-closed of Y is clopen in X. Noiri.T 1979 introduced the notion of perfect continuity between topological spaces. Recall that a function $f:(X,\tau)\to(Y,\sigma)$ is called perfectly continuous [Noire.T 1979] if the inverse image of every open set of Y is clopen in X.

Theorem 3.15. If the function$f:(X,\tau)\to(Y,\sigma)$ is ST-$\alpha\delta$-Continuous and the function $g:(Y,\sigma)\to(z,\eta)$ are $\alpha\delta$-continuous, then $g\circ f:(X,\tau)\to(z,\eta)$ is continuous.

Proof. Follows from Definitions 3.1 (a) and Definition 3.13.

Theorem 3.16. If $f:(X,\tau)\to(Y,\sigma)$ is perfectly continuous and if Y is both T^{δ}-space and $\widetilde{T_{\alpha\delta}}$-space, then f is PER-$\alpha\delta$-Continuous.

Proof. Follows from Theorem 2.25.

4.$\alpha\delta O$-compact spaces and αδ-connected spaces.

Definition 4.1. A topological space (X,τ) is called $\alpha\delta O$-compact if for every cover of X by $\alpha\delta$-open sets ($\alpha\delta$-open cover) has finite subcover.

Definition 4.2.[16] A topological space X is called πg-compact if every cover of X by πg-open sets has finite subcover.

Remark 4.3. Every πg-compact space is $\alpha\delta O$-compact.

Definition 4.4. A subset A of a space (X,τ) is called $\alpha\delta O$-compact relative to X if for every collection $\{\ U_i: i\in\Lambda\}$ of $\alpha\delta$-open subsets of X such that $A\subset\cup\{\ U_i: i\in\Lambda\}$, there exists a finite subset Λ_0 of Λ such that $A\subset\cup\{\ U_i: i\in\Lambda_0\}$.

Definition 4.5.A subset A of a space (X,τ) is called $\alpha\delta O$-compact if A is $\alpha\delta O$-compact as a subspace of$X$.

Lemma 4.6.A topological space (X,τ) is strongly compact if and only if X is compact and the Cantor-Bendixson derivative$X/I(X)$ is finite.

Example 4.7. Let $\boldsymbol{Z}$ be the set of all integers and τ be the double point excluded topology on $\boldsymbol{Z}$, *i.e.*,$\tau=\{U\subset\boldsymbol{Z}: U\cap\{0,1\}=\varphi\}\cup\{\boldsymbol{Z}\}$.Since this is a compact topological space with finite Cantor-Bendixson derivative, by Lemma 4.6.,$(\boldsymbol{Z},\tau)$is strongly compact. To see that $(\boldsymbol{Z},\tau)$ is not $\alpha\delta O$-compact, we consider the following αδ-open cover of $\boldsymbol{Z}$: $\mathcal{A}=\{\{1,x\}: x\in \boldsymbol{Z} \; and \; x\neq 0\}\cup\{0\}$. It is obvious that $\mathcal{A}$ has no finite subcover.

Theorem 4.8.

(a) Every $\alpha\delta$-closed subsets of a $\alpha\delta O$-compact space X is $\alpha\delta O$-compact relative to X.

(b) The surjective $\alpha\delta$-continuous image of a $\alpha\delta O$-compact space is compact.

(c) If $f:(X,\tau)\to(Y,\sigma)$ is αδ-irresolute and a subset A of X is $\alpha\delta O$-compact relative to X, then its image $f(A)$ is $\alpha\delta O$-compact relative to Y.

(d) If $f:(X,\tau)\to(Y,\sigma)$ is strongly αδ-continuous surjective and X is compact, then Y is αδO-compact.

Proof: (a) Let A be a αδ-closed subset of a $\alpha\delta O$-compact space (X,τ). Let $\{U_i: i\in\Lambda\}$ be a cover of A by αδ-open subsets of X. So $A\subset\bigcup_{i\in\Lambda}U_i$ and then $(X/A)\cup(\bigcup_{i\in\Lambda}U_i)=X$. Since X is αδO-compact, there exists a finite subset Λ_0 of Λ such that $(X/A)\cup(\bigcup_{i\in\Lambda_0}U_i)=X$. Then $A\subset\bigcup_{i\in\Lambda_0}U_i$ and hence A is αδO-compact relative to X.

(b) Let (X,τ) be a αδO-compact space and $f:(X,\tau)\to(Y,\sigma)$ be a surjective αδ-continuous function. Let $\{U_i: i\in\Lambda\}$ be a cover of X by open sets. Then $\{f^{-1}(U_i): i\in\Lambda\}$ is a cover of X by αδ-open sets, since f is αδ-continuous. By αδO-compactness of X, there is a finite subset Λ_0 of Λ such that $X=\bigcup_{i\in\Lambda_0}f^{-1}(U_i)$. Since f is surjective, $Y=\bigcup_{i\in\Lambda_0}U_i$ and hence Y is compact.

(c) and (d) are similar to (b).

Lemma 4.9. Let $p: X\times Y\to X$ be a projection. If A is αδ-closed subset of X, then $p^{-1}(A)=A\times Y$ is αδ-closed in $X\times Y$.

Proof. Let $A\times Y\subset U$ and U be a αg-open subset of $X\times Y$. Then $U = V\times Y$ for some αg-open set of X. Since A is αδ-closed in X $\delta\mathrm{cl}_X(A)\subset V$, and so $\delta\mathrm{cl}_X(A)\times Y\subset V\times Y=U$, *i.e.*, $\delta\mathrm{cl}_{X\times Y}(A\times Y)\subset U$. Hence $A\times Y$ is αδ-closed in $\times Y$.

Theorem 4.10. If the product space of two non-empty topological spaces is αδO-compact, then each factor space is $\alpha\delta O$-compact.

Proof. Let $X\times Y$ be the product space of the non-empty spaces X and Y and $X\times Y$ be $\alpha\delta O$-compact. By Lemma 4.9, the projection $p: X\times Y\to X$ is $\alpha\delta$-irresolute and then by Theorem 4.8(c), $p(X\times Y)=X$ is αδO-compact. The proof for the space Y is similar to the case of X.

Definition 4.11. A space (X,τ) is said to be αδ-connected if X cannot be expressed as the disjoint union of two nonempty αδ-open sets. A subset of X is αδ-connected if it is αδ-connected as a subspace.

Proposition 4.12. For a space (X,τ) the following are equivalent:

(a) X is $\alpha\delta$-connected.

(b) The only subsets of X which are both αδ-open and αδ-closed are the empty set φ and X.

(c) Every αδ-continuous function of X into a discrete space Y with at least two points is a constant function.

Proof. The proof is similar to that of Proposition 6.2 of [Gnanambal.Y et al. 1999].

Proposition 4.13.

(a) If (X,τ) is a space with $\tau^*_{\alpha\delta}=\tau$, then X is connected if and only if X is $\alpha\delta$-connected.

(b) If (X,τ) is a space with $\tau^*_{\alpha\delta}=\tau^{\#}_{g\delta s}$, then X is connected if and only if X is $g\delta s$-connected.

Proof.

(a) Follows from the definitions and Theorem 2.24 (b).

(b) Follows from the definitions and Theorem 2.24 (c).

Proposition 4.14.

(a) If $f:(X,\tau)\to(Y,\sigma)$ is $\alpha\delta$-continuous surjective and X is $\alpha\delta$-connected, then Y is connected.

(b) If $f:(X,\tau)\to(Y,\sigma)$ is $\alpha\delta$-irresolute surjective and X is $\alpha\delta$-connected, then Y is $\alpha\delta$-connected.

(c) Let $f:(X,\tau)\to(Y,\sigma)$ be a αδ-continuous surjection and $\alpha\delta C(X,\tau)$ is closed under finite intersections. If H is αg-open, $\alpha\delta$-closed and αδ-connected, then $f(H)$ is a connected.

Proof. (a) and (b) follow from definitions.

(c) By Proposition 3.10, the restriction $f|H$ of f is αδ-continuous. By (a), the image of the αδ-connected space $(H,\tau/H)$ under $f|H:(X,\tau|H)\to(f(H),\sigma|f(H))$ is connected. Hence $f(H)$ is connected subset of Y.

Theorem 4.15. If the product space of two non-empty spaces is $\alpha\delta$-connected, then each factor space is $\alpha\delta$-connected.

Proof. Let $X\times Y$ be the product space of non-empty spaces X and Y. By Lemma 4.9, the projection $p: X\times Y\to X$ is $\alpha\delta$-irresolute. By Proposition 4.14(b), the $\alpha\delta$-irresolute image $p(X\times Y)\,(=X)$ of the $\alpha\delta$-connected space $X\times Y$, is $\alpha\delta$-connected. The proof for the space Y is similar to the case of X.

Conclusion

In this paper we have studied about $\alpha\delta$-open and $\alpha\delta$-closed sets. Then we introduced $\alpha\delta$-irresolute and $\alpha\delta$-continuous functions via the concept of $\alpha\delta$-closed sets and to relate these concepts to the classes of $\alpha\delta O$-compact spaces and $\alpha\delta$-connected spaces.

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